Prediction of Pedestrians' Destinations based on the Theory of Probabilistic Projection Space

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***Abstract*—Pedestrian motion model is a research focus, involving a variety of realistic scenarios, such as transportation hub, shopping mall construction and evacuation, urban planning and large-scale event risk prevention. Its related multi-disciplines include physics, mathematics, psychology and management. The uncertainty of motion from pedestrian autonomy and complexity of real life scenes make it difficult to quantitatively calculate the movement of pedestrian, which limits our ability to establish pedestrian movement model. In current, there is not any universal model for describing pedestrian movement and for describing the impact of environment and pedestrian psychology on pedestrian movement. In paper, I use the theory of probability field and probabilistic projection space to establish a theoretical model of pedestrian movement in scenes with obstacles and destinations and to predict the destinations of pedestrians. Experimental results of a realistic scene verified that this model can accurately predict the destinations of pedestrians in general scenarios. I expect this descriptive model of pedestrian movement in complex space based on the theory of probability field and probability projection space can play a guiding role in calculation pedestrian motion and pedestrian management.**

Keywords*—Pedestrian Dynamic, Probability field, Probabilistic Projection Space, Field Theory, Prediction, Destination

I. INTRODUCTION

In many scenes of real life, we often need to calculate and analyze the movement and choice of pedestrians. Especially in large transport hubs, such as airports, subway stations and railway stations, the choice and movement of pedestrians are directly related to the safety of transport hubs. Pedestrian dynamic is a discipline describing the movement of pedestrians, involving physics, mathematics, psychology and management[1,2]. The uncertainty of pedestrians' independent choices, the complexity of real scenes, the time-varying environments and the complexity of interaction mechanics between pedestrians make it difficult for us to quantitatively calculate and predict pedestrians' motion[2,3]. Social force model[4,5,6], cellular automata model[7,8,9], discrete choice model[10,11], magnetic force model[12,13,14] and hydrodynamic model[15,16,17] are several existing pedestrian dynamics models, which can describe the characteristics of pedestrian movement to some extent or in specific scenarios, cannot consider the uncertainty of pedestrian self-selection and the rules of interaction between pedestrians, so the description of pedestrian movement in general scenarios is still not applicable. In addition, unlike fluids, electromagnetic objects and flocks, pedestrians have complex psychological activities[18,19,20,21,22]. At present, there is no basic model that can well import pedestrians' psychological activities into the dynamic model, which makes the assumptions of pedestrian dynamics do not fit well with the actual scenes. Therefore, the dynamic description of pedestrian motion is still a formidable challenge.

In order to accurately describe the motion model of pedestrians in general scenarios and establish the pedestrian stochastic dynamics model, I proposed the theory of probability field and probability projection space based on probability theory, geometric mapping and field theory. I use probabilistic projection space to describe pedestrian's motion selection in scenes, and establish a theoretical model of pedestrian's probabilistic projection space. Utilizing the concepts of probability field and probability projection space, the influence of obstacles and destination on pedestrian motion is established. In order to verify the feasibility and practicability of the model proposed in the paper, I have carried out experiments in a realistic scene to predict the destinations of the pedestrians. By comparing the predicted results calculated by the model proposed in the paper and the realistic results, the feasibility and applicability of the proposed model for describing pedestrian motion in complex scene with obstacles and destination based on probabilistic projection space are illustrated.

The paper is constructed as follows. In Section II, I introduce the theory of probability field and probabilistic projection space, which is proposed by the author of this paper. In Section III, I establish a model of probabilistic projection space for a single pedestrian in space. In Section IV, I verify the practicality and accuracy of model to predict the destinations of pedestrians through experiments.

II. THEORY OF PROBABILITY FIELD AND PROBABILISTIC PROJECTION SPACE

In order to facilitate the establishment of pedestrian dynamics model, this section will briefly introduce the probability field and probabilistic projection space theory proposed by the author. The theory of probability field and probabilistic projection space originates from the combination and innovation of probability theory, space geometry and field theory[23,24,25] aimed at describing the complex stochastic process of many kinds of fields with probability theory and field theory. This theory proposes a more universal model for the stochastic process analysis of various complex stochastic systems such as the flow field, the electromagnetic field and the gravitational field. The research in this paper is an application of the theory of probability field and probabilistic projection space in pedestrian dynamics model.

A. Theory of Probability field

The definition of probability field is as Definition 1.

Definition 1. Assuming the aggregate of a region is U, the probability density function of $x \in U$ is $f(x)$, and the probability density function $f(x)$ meets that $\int_{x\in U} f(x)dx = 1$. If the probability density of some region A in the aggregate U is limited or adjusted, the probability density distribution of the aggregate U will be changed. The region aggregate U and probability density function $f(x)$ satisfying this property constitute a continuous probability space marked as $Z_\alpha\left(U, f(x)\right)$ in this paper, which can be defined as a continuous probability field about U and $f(x)$.

The probability field has following properties.

Property 1. The probability sum in aggregate U is 1.

Property 2. The existence of a certain factor will cause the probability of some parts of the aggregate to change. There are two factors causing the change: boundary conditions and Disturbance.

Property 3. If the probability of a part changes, the probability of the other part will change correspondingly, that is, there is a certain coupling relationship within the probability field.

Property 4. The basic elements of a probability field are the set of value regions and the corresponding probability density function or probability, that is, the probability field is uniquely determined by the set of value regions and the probability distribution.

About probability field, I have defined two factors based on the properties of field theory that is the boundary condition as Definition 2 and disturbance as Definition 3.

Definition 2. The probability distribution of the probability field will be affected by a certain restriction on the value region of probability field. This restriction on the value region is called the boundary condition of the probability field.

Definition 3. The probability distribution of a probability field will be affected if the probability of some region is limited. Such a restriction on the probability of some regions is called the disturbance of probability field.

A typical boundary condition that is fixed boundary condition is defined as Definition 4.

Definition 4. When the region corresponding to the boundary condition remains unchanged, if the original random distribution remains unchanged when the n-th experiment falls outside the valid region limited by the boundary condition, the experiment will continue until it falls within the valid region limited by the boundary condition. Such boundary conditions are called fixed boundary conditions.

Then, the fixed boundary condition satisfies Theorem 1.

Theorem 1. For an arbitrary probability field $Z(U, F)$, the relation between it and the new probability field $Z_A^D(U, F)$ generated by adding a fixed boundary condition with region A is as

$$Z_A^D(U,F) = Z\left(U - A, \frac{F(x)}{\sum_{x\in(U-A)} F(x)}\right). \tag{1}$$

B. Theory of Probability field

Mapping of probability field can be defined as

Definition 5. Mapping refers to the corresponding relationship between two aggregates of elements. It refers to the correspondence f between two non-empty aggregates A and B. For each element x in A, there is always a unique element y in B that corresponds to it. This correspondence is called a mapping from A to B, which is denoted by $f: A \to B$. When the element of aggregate A and the element of aggregate B are probability fields, the mapping relation is called the mapping of probability fields. For the element x in the aggregate, another probability field y will be generated after mapping. If the elements of a non-empty aggregate A are all probability fields, such a set is called an aggregate of probability fields.

Then, probabilistic projection space can be defined as Definition 6.

Definition 6. If the original image in an aggregate A is one-to-one with the image in the mapping aggregate B, where $f: A \to B$ and the probability field of the original image in A is $Z\left(A, g(x)\right)$, then the probability field of the image of mapped in B is $Z\left(B, f^{-1}(x)g\left(f^{-1}(x)\right)\right)$. Conversely, if the probability field of the image of mapped in B is $Z\left(B, h(x)\right)$, then the probability field of the original image in A is $Z\left(A, f(x)h\left(f(x)\right)\right)$. This space containing the probability field of original image and the probability field of mapping image can be called a probabilistic projection space.

The probabilistic projection space satisfies Theorem 1 that can be called Projection Flux Theorem of Probability Field.

Theorem 2. If the original image in an aggregate A and the image in its mapping aggregate B satisfy the relation $f: A \to B$, where the probability field in the original image is $Z\left(A, f_A(x)\right)$, and the probability field in the projection space is $Z\left(B, f_B(x)\right)$. If any region A_1 in the original image space is projected to the image space as the region B_1, then the following relationship is true.

$$\int_{A_1} f_A(x)dx = \int_{B_1} f_B(x)dx \tag{2}$$

The probabilistic projection vector which can show the probabilistic intensity of a point in probabilistic projection space can be defined as Definition 7.

Definition 7. The vector composed of the projection direction and the corresponding probability intensity of a point in the probability field projection space is called the probabilistic projection vector. A n dimensional probabilistic projection space $\mathbb{R}^n$ can be represented by its probabilistic projection vector as $\Upsilon(\Omega, \boldsymbol{E})$ where Ω is the range of values in probabilistic projection space and $\boldsymbol{E}$ is the vector function of probabilistic projection vector.

Three types boundary conditions probabilistic projection space that is probabilistic generator, probabilistic absorber and probabilistic insulator can be defined as Definition 8.

Definition 8. If an object can project a probabilistic projection vectors, it is called probabilistic generator. If the probabilistic projection cannot be projected on a certain object in a probabilistic projection space, which means the probability flux of the projection vector reaching the object is 0. Since the projection vector cannot pass through any surface of the object, the projection vector on the surface of the object will be parallel to the surface of the object. Similarly, the probabilistic projection space will be affected if the probability field of an object's surface absorption is given when a certain probability is required. Because it absorbs some probability, the probability vector on its surface is always perpendicular to its surface. Such an object is called a probability absorber.

The factors of probability projection space have the theorem as Theorem 3.

Theorem 3. If the probability generator, probability receiver, boundary condition, disturbance and space shape change in the probabilistic projection space, the probabilistic projection vector in the space will change. In space, any closed curve containing all probabilistic generators is a probability field and the flux of probabilistic projection vector at a point on the curvy is the probability density of a probability field.

For a translational probabilistic generator, the probability distribution of a certain probability field will change with the movement of the probabilistic generator in the probabilistic projection space. And then, time probability field can be obtained as Definition 9.

Definition 9. If the probability field $Z(U, F)$ varies with time, that is, the probability field contains variables of time, then the probability field is called the probability field of time domain. The expression of the probability field for a time series is $Z(U(t), F(x,t))$.

III. Model of Probabilistic Projection Space for Single Pedestrian in Space

Based on the theory of probability field and probability projection space, we can regard a pedestrian in planar space as a movable probability generator that can project the probability projection vector with probability of 1. The direction of the probabilistic projection vector is the pedestrian's walking path, and the size of the probability projection vector is the probability intensity of the pedestrian's walking. In addition, the probability projection space generated by pedestrians will change with the movement of pedestrians.

It can be assumed that a pedestrian is a circular with a certain radius r in planar space. In an infinite planar space without any restriction of environmental factors, the probability projection space of a pedestrian without subjective consciousness at point $P(t) = (x(t), y(t))$ at time t which has to move all the time is a uniformly divergent probability projection space as $\Upsilon\left(\forall A \in \mathbb{R}^2, \boldsymbol{E}_A = \boldsymbol{PA} / \left(2\pi |\boldsymbol{PA}|^2\right)\right)$ whose diagram is as Fig. 1. It can be known that the equal probability curvy of $\Upsilon\left(\forall A \in \mathbb{R}^2, \boldsymbol{E}_A = \boldsymbol{PA} / \left(2\pi |\boldsymbol{PA}|^2\right)\right)$ is circles centered on P.

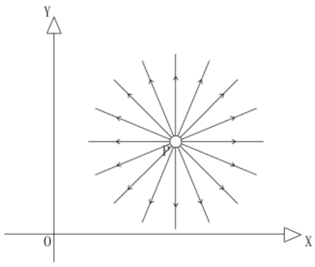

Fig. 1. Diagram of the probabilistic projection space that is $\Upsilon\left(\forall A \in \mathbb{R}^2, \boldsymbol{E}_A = \dfrac{\boldsymbol{PA}}{2\pi \boldsymbol{PA}^2}\right)$.

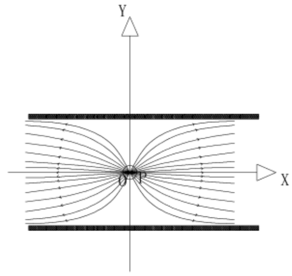

Fig. 2. Diagram of the probabilistic projection space with two infinite parallel obstacles $y = l$ and $y = -l$.

Based on this, I will model the movement of pedestrians in space.

A. Probabilistic projection space model of single pedestrian in space with obstacles

In order to research the effect of the space environments on the motion of pedestrian, I set obstacles in the space where pedestrians are located.

In planar space, an obstacle whose surface is S can be regarded as a probabilistic insulator, where the probabilistic projection vector cannot pass through it.

It can be known $\exists \varphi(x, y)$ *st.* $grad\left(\varphi(x, y)\right) = \boldsymbol{E}(x, y)$ where $grad(\varphi)$ is the gradient of φ. Then in the probabilistic projection space, φ must satisfy

$$\Delta\varphi(x, y) = 0 \tag{3}$$

where $(x, y) \in \mathbb{R}^2$. On the surface of obstacle S, φ must satisfy

$$\frac{\partial\varphi(x, y)}{\partial \boldsymbol{n}_S} = 0 \tag{4}$$

where $(x, y) \in S$ and $\boldsymbol{n}_S$ is the normal vector of surface S. For any closed curves arbitrarily containing all of the probabilistic generators, φ must satisfy

$$\oint_C \frac{\partial \varphi(x,y)}{\partial \boldsymbol{n}_C} = P_{sum} \tag{5}$$

where $\boldsymbol{n}_C$ is the normal vector outward of C and P_{sum} is the sum of the probabilities of all probability generators. Based on Eq. 3, Eq. 4 and Eq. 5, we can get the probabilistic projection vector of the pedestrian in a planar space with obstacles. For example, we can get the diagram of the probabilistic projection space and the probabilistic projection vector of the pedestrian in a space with two infinite obstacles as Fig. 2, Fig. 3 and Fig. 4.

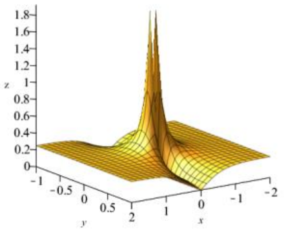

Fig. 3. Three dimensional diagram of probability density of the probability field on $x = \pm const$ for $0 < const < +\infty$ and $l = 1$ in probabilistic projection space. $-2 \le x \le 2$, $-1 \le y \le 1$ and z is the probability intensity of probability field.

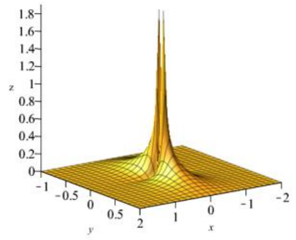

Fig. 4. Three dimensional diagram of probability density of the probability field on $y = \pm const$ for $0 < const \le l$ and $l = 1$ in probabilistic projection space. $-2 \le x \le 2$, $-1 \le y \le 1$ and z is the probability intensity of probability field.

B. Probabilistic projection space model of single pedestrian with destination

In order to research the effect of the space environments on the motion of pedestrian, I set destinations for the single pedestrian. Destinations can be regarded as a subjective consciousness of pedestrian, which is an origin of pedestrians' psychological factors.

In planar space, a destination whose surface is D can be regarded as a probabilistic absorber, which forces the absorption probabilistic projection vector so that the probability on its surface equals its own probability intensity.

A destination that is a probabilistic absorber in probabilistic projection space will produce probabilistic projection vectors pointing at it which will change the original probability space of pedestrian. For any closed curves arbitrarily containing all destinations, φ must satisfy

$$\oint_C \frac{\partial \varphi(x,y)}{\partial \boldsymbol{n}_C} = -P_{sum} \tag{6}$$

where $\boldsymbol{n}_C$ is the normal vector outward of C and P_{sum} is the sum of the probabilities of all destinations. According to the probability superposition property of destinations, we can get the following theorem about destinations.

Theorem 4. Assuming there is n absorptions whose probability intensity is α_i where $i \in \mathbb{N}^*$, $i \in [1,n]$ and $\sum_{i=1}^{n} \alpha_i = 1$, and assuming the probabilistic projection space generated by the i-th absorbers is $\Upsilon\left(\mathbb{N}^*, \boldsymbol{E}_i\right)$ when the probability intensity of each absorbers is 1, then the probabilistic projection space of n absorbers is $\Upsilon\left(\mathbb{N}^*, \sum_{i=1}^{n} \alpha_i \boldsymbol{E}_i\right)$.

For a point destination D without any environmental conditions which can be regard as a circular with a certain radius r in planar space, the probabilistic projection space generated by D is also a uniformly divergent probability projection space as $\Upsilon\left(\forall A \in \mathbb{R}^2, \boldsymbol{E}_A = \boldsymbol{AD} / \left(2\pi \,|\, \boldsymbol{AD} \,|^2\right)\right)$ pointing to D whose diagram is as Fig. 5.

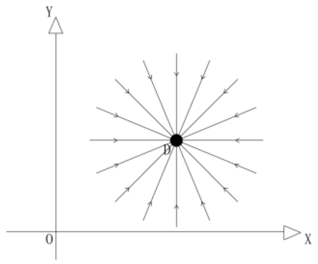

Fig. 5. Diagram of the probabilistic projection space $\Upsilon\left(\forall A \in \mathbb{R}^2, \boldsymbol{E}_A = \dfrac{\boldsymbol{AD}}{2\pi \boldsymbol{AD}^2}\right)$.

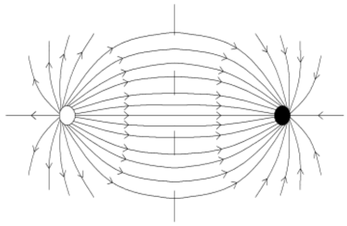

Fig. 6. Diagram of the probabilistic projection space $\Upsilon\left(\forall A \in \mathbb{R}^2, \boldsymbol{E}_A\right)$ and $\boldsymbol{E}_A = \dfrac{1}{4\pi}\left(\dfrac{\boldsymbol{AD}}{\boldsymbol{AD}^2} + \dfrac{\boldsymbol{PA}}{\boldsymbol{PA}^2}\right)$.

For destination with complex shape, the probabilistic projection space generated by it can be seen as the vector superposition of probabilistic projection vectors formed by infinite points in its surface region. It can be set that the surface region D of destination whose probability intensity is P_D can be evenly decomposed into n points where $n \to +\infty$ and each point's probability intensity is P_D / n. When the probabilistic projection vector at point A of i-th point K_i is set as $\boldsymbol{E}_i(A)$, then $\exists \varphi_i$ *st.* $grad\left(\varphi_i(A)\right) = \boldsymbol{E}_i(A)$ for $\forall A \in \mathbb{R}^2$. Thus, φ_i must satisfy Eq. 3, Eq. 7 and Eq. 8.

$$\oint_C \frac{\partial \varphi(x,y)}{\partial \boldsymbol{n}_C} = -\frac{P_D}{n} \tag{7}$$

where $\boldsymbol{n}_C$ is the normal vector outward of any closed curves C arbitrarily containing i-th point.

$$\oint_C \frac{\partial \varphi(x,y)}{\partial \boldsymbol{n}_{D_i}} = 0 \tag{8}$$

where $(x,y) \in D - K_i = D_i$ and $\boldsymbol{n}_{D_i}$ is the normal vector of surface D_i.

Therefore, we can get that the probabilistic projection space whose probabilistic projection vector is $\boldsymbol{E}_A$ generated by the destination with complex shape must satisfy that

$$\varphi = \lim_{n\to\infty} \frac{P_D}{n} \sum_{i=1}^{n} \varphi_i \tag{9}$$

where $grad(\varphi) = \boldsymbol{E}_A$.

Adding pedestrian to space with destination, if pedestrian's probability projection vector is $\boldsymbol{E}_{PA}$ and destinations' probability projection vector is $\boldsymbol{E}_{DA}$ at the point A, the probability projection vector of the coincident probability projection space generated by the pedestrian and destinations is $\boldsymbol{E}_{PA} + \boldsymbol{E}_{DA}$ at the point A.

Then for a planar space with a pedestrian without subjective consciousness at point P and a point destination D without any environmental conditions, the probability projection space generated by them is $\Upsilon\left(\forall A \in \mathbb{R}^2, E_A = \frac{1}{2\pi}\left(\frac{\boldsymbol{AD}}{\boldsymbol{AD}^2} + \frac{\boldsymbol{PA}}{\boldsymbol{PA}^2}\right)\right)$ whose diagram is as Fig. 6.

In order to accurately describe the movement of pedestrians in real environment, we need to integrate the probabilistic projection space models of pedestrian with and destinations and obstacles.

Depending on the properties of the probabilistic projection space, the probabilistic projection vectors generated by the destination and pedestrians will be affected by obstacles. Assuming the pedestrian is at the point P, the destination is D and obstacle is S, then the probabilistic projection space generated by the pedestrian, destination and obstacle must satisfy Eq. 3, Eq. 4, Eq. 5 and Eq. 6.

Similarly in this way, we can apply the pedestrian model based on the theory of probability field and probabilistic projection space to the realistic scene for analysis of pedestrian movement law. Next section, I will validate the practicability and accuracy of the model to predict the destinations of pedestrians through the realistic scene experiments.

IV. Prediction Experiments of the Pedestrians

A. Prediction Experiment of Pedestrians' Destinations in a Cinema

Based on the theory of probabilistic projection space, we can utilize the model of probabilistic projection space to predict the pedestrians' destinations according to the positions of pedestrians.

In order to predict the pedestrians' destinations, we could give a theorem of conditional probability as Theorem 5 based on Theorem 2.

Theorem 5. For probability fields $Z(U_1, F_1)$ and $Z(U_2, F_2)$ with arbitrary projection rules, let event X be that the value of probability field $Z(U_1, F_1)$ is on region $A_1 \subset U_1$ and event Y be that the value of probability field $Z(U_2, F_2)$ is on region $A_2 \subset U_2$. The projection of region A_1 on U_2 is B_1 and the projection of region A_2 on U_1 is B_2. Then, the relationship of conditional probability is as

$$\begin{cases} \mathrm{P}(Y \mid X) = \dfrac{\sum_{x\in A_1\cap B_2} F_1(x)}{\sum_{x\in A_1} F_1(x)} = \dfrac{\sum_{x\in B_1\cap A_2} F_2(x)}{\sum_{x\in B_1} F_2(x)} \\ \mathrm{P}(X \mid Y) = \dfrac{\sum_{x\in A_1\cap B_2} F_1(x)}{\sum_{x\in A_2} F_1(x)} = \dfrac{\sum_{x\in B_1\cap A_2} F_2(x)}{\sum_{x\in B_2} F_2(x)} \end{cases} \tag{10}$$

Based on Theorem 5, we can research a simple example.

Example 1. It can be assumed that the possible destination of pedestrian is a circle with radius $2R$, the starting point of pedestrian is the origin of coordinates and the coordinate of pedestrians on a circle with the origin as the center and radius of R are $(R\cos\theta, R\sin\theta)$. The diagram is as Fig. 7.

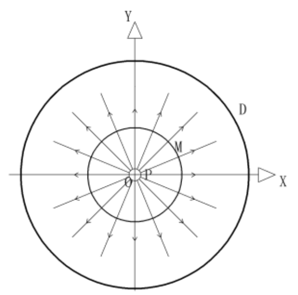

Fig. 7. Diagram of the space with a circle destination.

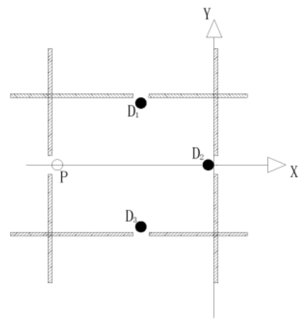

Fig. 8. Diagram of cinema passageway with three viewing halls.

When the pedestrian destination coordinate is $(2R\cos\theta_1, 2R\sin\theta_1)$, the probability density of the pedestrian's probability field on the circle with radius R at point $(R\cos\theta, R\sin\theta)$ is $\dfrac{2-\cos(\theta-\theta_1)}{\pi R\left(5-4\cos(\theta-\theta_1)\right)}$. According to Eq. 10, we can obtain that the probability of pedestrian's destination is $P\left((2R\cos\theta_1, 2R\sin\theta_1) \mid (R\cos\theta, R\sin\theta)\right) = \dfrac{2-\cos(\theta-\theta_1)}{\pi R\left(5-4\cos(\theta-\theta_1)\right)}$.

Ex. 1 has explained the idea of predicting pedestrian's destination through the pedestrian's location based on the model of probabilistic projection space. Thus, we can design experiments depending on this idea.

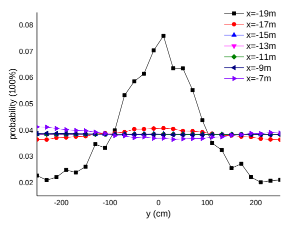

Fig. 9. Diagram of statistical probability of each cross-sections in the experiments of cinema.

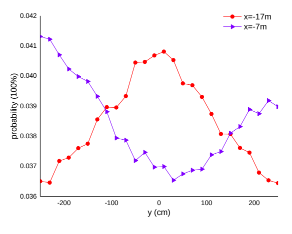

Fig. 10. Diagram of statistical probability of cross-sections $x = -17m$ and $x = -7m$ in the experiments of cinema.

I conducted experiments to verify the feasibility of the model in predicting the destinations of pedestrians at the passage of a cinema. By recording the videos and extracting pedestrian coordinates from the videos, I carried out the experiments and got the walking routes of several thousand individual pedestrians long before the opening of each movie so that the pedestrians' walking will not be affected by other pedestrians. During the dates of experiments, the cinema only opened three viewing halls and the diagram of its passageway is as Fig. 8. In Fig. 8, the entrance of the passageway is at point $P=(-20m,0)$ and the entrances of the three viewing halls are respectively $D_1=(-5m,2.5m)$, $D_2=(0,0)$ and $D_3=(-5m,-2.5m)$. I intercepted seven cross-sections $x=-19m$, $x=-17m$, $x=-15m$, $x=-13m$, $x=-11m$, $x=-9m$, $x=-7m$, and then I recorded the longitudinal coordinate data of the pedestrians passing through the cross-sections. The statistical data is as Fig. 9.

In order to observe the law easily, we can put the same order of magnitude of diagrams together as Fig. 10 and Fig. 11. With the statistical probability of each cross-section, we can predict the probability of pedestrians entering each viewing hall based on Theorem 5 through the iterative extraction of parameters, and the predicted results are as Table I and Fig. 12 to Fig. 15 where the real ratios of pedestrians in each viewing hall are $P(D_1)=0.2137$, $P(D_2)=0.4968$ and $P(D_3)=0.2895$.

TABLE I. THE RESULTS OF PREDICTION FOR THE EXPERIMENTS OF CINEMA WITH THREE DESTINATIONS.

Destination	Reality	$x=-19m$	$x=-17m$	$x=-15m$	$x=-13m$	$x=-11m$	$x=-9m$	$x=-7m$
D_1 hall	0.2137	0.1683	0.1692	0.1803	0.1929	0.2016	0.2063	0.2093
D_2 hall	0.4968	0.5125	0.5147	0.5144	0.5134	0.5083	0.5100	0.4994
D_3 hall	0.2895	0.3192	0.3161	0.3053	0.2937	0.2901	0.2837	0.2913

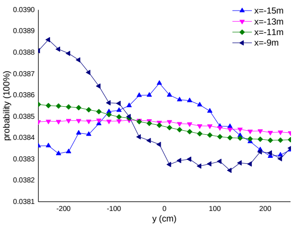

Fig. 11. Diagram of statistical probability of cross-sections $x=-15m$, $x=-13m$, $x=-11m$ and $x=-9m$ in the experiments of cinema.

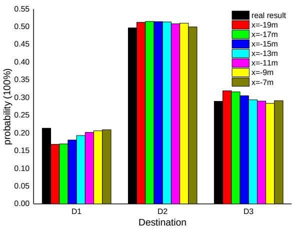

Fig. 12. Diagram of predicted probability of pedestrians' destinations.

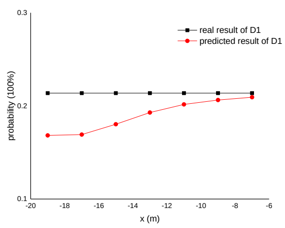

Fig. 13. Diagram of predicted probability of pedestrians to D_1 hall.

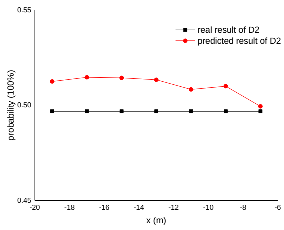

Fig. 14. Diagram of predicted probability of pedestrians to D_2 hall.

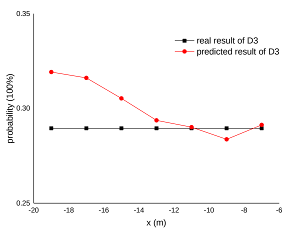

Fig. 15. Diagram of predicted probability of pedestrians to D_3 hall.

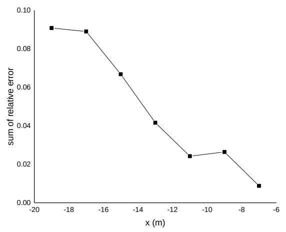

Fig. 16. Sum of relative error between the predicted result and realistic result of three destinations.

From the results of Table I and Fig. 12 to Fig. 15, it can be known that the error between the predicted results and the real results is less than 5%. The predicted results according to the cross-section $x=-19m$ can reach a high accuracy. As the cross-section approaches the destinations, the accuracy of the prediction becomes higher and higher. And for the cross-section $x=-7m$, the error between the predicted results and the real results is less than is less than 0.5%. In order to visualize the trend of error, we can obtain the relationship between relative error and cross-sections as Fig. 16.

The results of Fig. 16 reiterated that the closer the cross-section is to the destination, the more accurate the predictions are. And the range of the sum of relative error is in the $(0.88\%, 9.08\%)$, which shows that accuracy of the model based on the probabilistic projection space is as high as $90\%-99\%$. This accuracy shows that our model proposed in the paper can more accurately restore the realistic path of pedestrians in the environment without interference of other pedestrian. The experiments have verified the feasibility, practicability, accuracy and superiority of the model proposed in the paper, which shows that the model can be used as a basic model for further research of pedestrian dynamic. In the follow-up research, we would like to add the velocity vector of pedestrian motion to predict pedestrians' destinations, which may improve the prediction accuracy and make the prediction through a cross-section far from the destination also achieve high accuracy.

B. Prediction Experiment of Pedestrians' Destinations in an Airport

In order to illustrate the practicability of the model for prediction of the pedestrians' destination in the airport, I conducted an observation experiment on the passageway of an airport. The diagram of the scene is as Fig. 17.

In Fig. 17, there are two directions that pedestrians can choose. Thus, we can regard the corner of the two directions as two destinations D_1 and D_2. Through the measurement, we can get that $P=(-15m,0)$, $D_1=(0,2.5m)$ and $D_2=(0,-2.5m)$. By recording the videos and extracting the pedestrians' coordinates from the videos, I carried out the experiments and got the walking routes of several thousand individual pedestrians. Similar to the experiments in Section IV-A, I interpreted four cross-sections $x=-12m$, $x=-9m$,

$x = -6m$, $x = -3m$ and recorded the coordinates of the pedestrians on the cross-sections, where the statistical results are as Fig. 18.

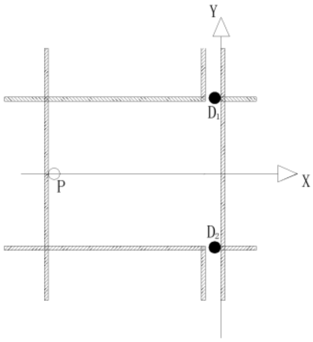

Fig. 17. Diagram of airport with two destinations.

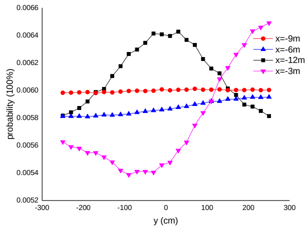

Fig. 18. Diagram of the statistical probability of each cross-section in the experiments of airport.

In order to observe the law easily, we can put the same order of magnitude of diagrams together as Fig. 19. With the statistical probability of each cross-section, we can predict the probability of pedestrians entering each direction based on Theorem 5 through the iterative extraction of parameters, and the predicted results are as Table II, Fig. 20 and Fig. 21 where the real ratios of pedestrians in each viewing hall are $P(D_1) = 0.3947$ and $P(D_2) = 0.6053$. Then, we can get the sum of relative error between predicted results and the realistic results as Fig. 22, where the closer the cross-section is to the destination, the more accurate the predictions are.

TABLE II. THE RESULTS OF PREDICTION FOR THE EXPERIMENTS OF CINEMA WITH THREE DESTINATIONS.

Destination	Reality	$x = -12m$	$x = -9m$	$x = -6m$	$x = -3m$
D_1 direction	0.3947	0.3569	0.3795	0.3824	0.3900
D_2 direction	0.6053	0.6431	0.6205	0.6176	0.6100

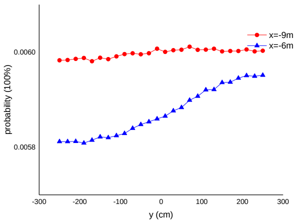

Fig. 19. Diagram of statistical probability of the cross-sections $x = -9m$ and $x = -6m$ in the experiments of airport.

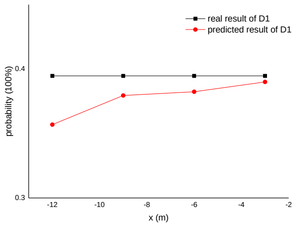

Fig. 20. Diagram of predicted probability of pedestrians to D_1 direction in the experiments of airport.

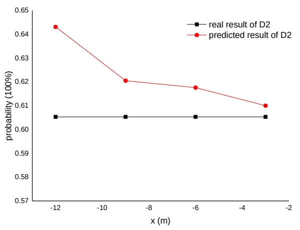

Fig. 21. Diagram of predicted probability of pedestrians to D_2 direction in the experiments of airport.

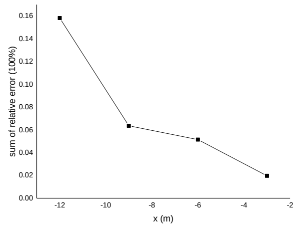

Fig. 22. Sum of relative error between the predicted result and realistic result in the experiments of airport.

The results of the experiments not only verify the practicability and feasibility in prediction of the pedestrians' destination again, but also illustrate the applicability of the model presented in this paper in transportation hub such as airport.

V. CONCLUSION

This paper describes the probability law of pedestrian movement based on the theory of probability field and probability projection space. The theory of probabilistic projection space is helpful to describe the influence of pedestrian's psychological and environmental factors on their movement, which is s conducive to our management of pedestrian movement. Based on the theory of probabilistic projection space, this paper presents a probabilistic calculation model for pedestrians' movement in realistic scenes with obstacles and destinations. In the paper, I also propose a method to predict the destination of the pedestrian based on the theory of probabilistic projection space. Through experiments, I verified the feasibility and accuracy of the model in prediction of the pedestrians' destinations. The accurate prediction of pedestrians' destinations is of great guiding significance for pedestrian guidance, safety control of pedestrian, as well as building structure and layout of large-scale transport hub. The theory and ideas in this paper are also helpful to establish a complete model of pedestrian stochastic dynamics, which has far-reaching significance for the development of pedestrian dynamics.

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The author only observed the walking process of pedestrians and recorded the data in the experiments. The experiments in this paper were carried out under the condition of zero interference from the observers to the pedestrians.

All of the theory in paper is proposed by the first author (Guangyao Zhou), and the research in paper is completed by the first author. The experiments in this paper are completed independently by the first author at his own expense.

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